

# Gluonic Structures of Tetraquarks

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## Abstract

We formulate quantitative flux-tube overlap functions to account for the nonperturbative gluonic behaviors. In order to deduce systematic functional form, we introduce connection amplitude which can be defined between two boundary points occupied by quark or antiquark. With the deduced flux-tube overlap function, we can figure out long range gluonic structures of various quark combinations. In this paper, we report our calculated results for tetraquarks,  $q^2\bar{q}^2$ , by considering two different configurations of boundary points for possible explanations of  $X(3872)$ .

## 1 Introduction

The quantum numbers of hadrons such as mesons and baryons are usually described by combining those of valence quarks. However, from the dynamical viewpoint, the binding forces are generated by gluons. These gluons not only generate binding forces, but also constitute important parts of hadrons affecting the long range structures. For the simplest meson system, mass spectra can be estimated by considering only one gluon exchange diagrams [1], whereas the next complicated baryon system cannot be easily analyzed without introducing gluonic degrees of freedom [2]. Even for the meson system, strong decay processes cannot be accounted appropriately without considering gluonic effects [3]. These gluonic effects are in the realm of nonperturbative interactions, which can only be described by phenomenological models such as flux-tube model.

Gluonic flux-tube picture was introduced to put the gluonic degrees of freedom explicitly into the calculations of baryon spectra [2], and later, quantitative approach was made to predict strong decays of mesons [3]. Strong decays of hadrons are induced by quark pair creations which are closely related to the gluon densities or gluonic energies. Since the position of created quark pair affects the final states of strong decay, it is important to check the gluonic profiles of initial state. For mesons, it was firstly attempted to introduce cigar-shape profiles, which were changed into more generalized forms including the case of spherical shape [4]. The consideration of spherical shape of gluonic flux-tube is motivated by the calculational convenience in treating quark wave functions. Strong decay amplitudes are dependent on the product of quark wave functions and gluonic flux-tube overlap functions, and therefore it is convenient to use the same form of function for the quarks and the gluons if possible. One possible choice is the harmonic wave functions which can be used safely for quark motions and the Gaussian form can be applied to describe gluonic

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behaviors. Because the Gaussian form is a function of radius, the resulting gluonic structure becomes that of spherical shape. However, it is quite arbitrary whether we choose the cigar shape or the spherical shape. Each choice has its merits and defects, and there existed no consensus on how to fix the form of flux-tube overlap function.

In order to deduce systematically the form of flux-tube overlap function, we need to analyze the structures of flux-tubes and to introduce some quantitative function that fits to the description of those structures. We can start the analysis by classifying the flux-tubes resulting in the natural consideration of various structures such as tetraquarks, pentaquarks, hexaquarks, and so on. The relations between these structures can be established by taking account of quark pair creations and quark pair annihilations with the effects of flux-tube breakings and connections respectively. The related flux-tubes can be shown to form topological spaces [5], and we can define physical amplitudes on the closed topological spaces [6]. For the physical amplitude, we are seeking for something that can be used to describe gluonic states. Since the long range gluonic states are described by flux-tubes, the connectedness through given flux-tube between two quarks can be chosen to define appropriate measure that can be used to estimate gluonic states. In this way, a systematic definition of flux-tube overlap function can be made and applied easily to various situations such as tetraquarks and pentaquarks in addition to the ordinary mesons and baryons [7].

The tetraquarks and pentaquarks have peculiar positions between ordinary hadrons and nuclei. For mesons and baryons, there exist so nice models to account for their mass spectra and decay processes that their quark compositions as  $q\bar{q}$  and  $qqq$  can never be replaced by other ones. On the other hand, stable nuclei can be explained more appropriately by the bound states of nucleons rather than by quarks and gluons. Even the simplest nucleus, deuteron, cannot be easily described by quarks and gluons. Therefore it is quite important to check whether we can extend the descriptions by quarks and gluons to more complicated systems. In this respect, tetraquarks and pentaquarks are good candidates to be studied by using quarks and gluons. Recent discoveries of  $X(3872)$  [8] and  $\Theta^+(1540)$  [9] stimulated the concerns about the structures of these states. However, as is well-known, the pentaquarks were predicted in the context of chiral soliton model rather than in the picture of five quarks bound by gluons [10]. The problems of 4 or 5 bodies are quite difficult to treat, and moreover, the non-linear gluonic interactions have to be considered via new method such as flux-tubes.

In this paper, we will devise a new method to set up flux-tube overlap functions and apply them to tetraquark configurations. Since we don't know the positions of quarks, we will consider two cases, one in the form of tetrahedron and the other in rectangular shape. We can calculate gluonic structures for each cases which can be compared quantitatively resulting in the selection of lowest energy configuration. In section 2, we will give general formalism, and calculated results are given in section 3. The final section is devoted to discussions and further problems.

## 2 Flux-tube Formalism

The flux-tube picture has been introduced to account for the long range behaviors of gluons. Here the long range means that of near 1 fermi, and for smaller region than 0.1 fermi we can apply perturbative methods to calculate physical amplitudes. Therefore flux-tube formalism is useful for the ranges between 0.1 fm and 1 fm. Since ordinary hadrons are estimated to have the extensions of 1 fm, flux-tubes can be applied at least to the descriptions of mesons and baryons. In addition, we can generalize the structures of flux-tubes to include many quark systems such as tetraquarks, pentaquarks, hexaquarks [11],

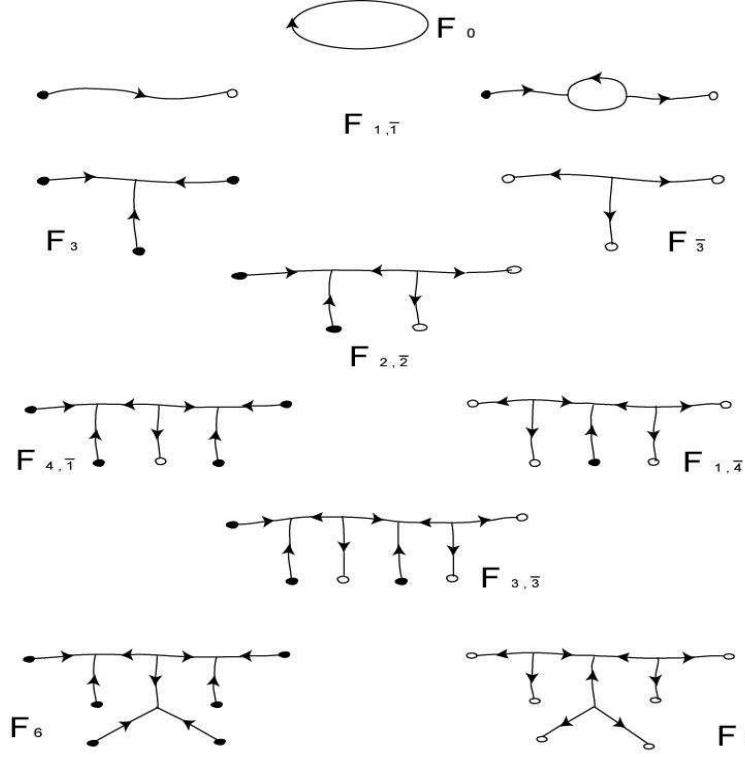


Figure 1: The classified flux-tubes.  $F_{2,\bar{2}}$  represent tetraquarks and  $F_{4,\bar{1}}$  and  $F_{1,\bar{4}}$  represent pentaquarks.

and so on. These generalizations are closely connected to the classification scheme of flux-tubes.

In order to classify flux-tubes, we need to count the number of boundary points on which quarks or antiquarks sit. We can take the flux-tubes as starting from quark boundaries and ending at antiquark boundaries. Then we can represent the set of flux-tubes with  $a$  quarks and  $b$  antiquarks sitting at boundaries as  $F_{a,\bar{b}}$ . Mesons are represented by  $F_{1,\bar{1}}$ , and baryons correspond to  $F_{3,\bar{0}}$  with  $F_{0,\bar{3}}$  that of antibaryons. In general, we can omit the number 0 without confusion so that  $F_3$  represents baryon flux-tubes and  $F_{\bar{3}}$  corresponding to antibaryons. Exceptionally  $F_{0,\bar{0}}$  can be reduced to  $F_0$  which represents the flux-tubes of glueballs. In this notation, only different topological configurations are distinguished and therefore different energy states with the same topology belong to the same set of flux-tubes. The classified flux-tubes are shown in Fig.1 where we can find naturally the structures of tetraquarks, pentaquarks, hexaquarks, and so on.

Now let's consider relationships between flux-tubes. The topological structure of a given flux-tube can be changed only when the number of boundary points is changed. The changes of boundary points are induced by quark pair creations and quark pair annihilations. If one quark pair is created in a flux-tube, the tube will be divided into two, and conversely two flux-tubes can be united into one through quark pair annihilation. The changes of division and union can be combined to establish relationships between flux-tubes. The most general relationship turns out to be the construction of topological spaces of flux-tubes. For the construction of topological spaces, we need to write down the following assumptions :

1. Open sets are stable flux-tubes.
2. The union of stable flux-tubes becomes a stable flux-tube.
3. The intersection between a connected stable flux-tube and disconnected stable flux-tubes is the reverse operation of the union.

These assumptions are based on the correspondence between flux-tubes and topological open sets, and the operations of union and intersection are quite natural with respect to the changes of flux-tubes. If we find out closed sets under the operations of union and intersection, we can classify flux-tubes into different topological spaces. In fact, this classification can be made by counting the numbers of incoming and outgoing 3-junctions in a given closed set.

For examples, let's consider first the flux-tubes  $F_{1,\bar{1}}$  corresponding to simple quarkonium mesons. If quark pair creations are repeated disconnecting the flux-tubes, we get in general  $n$  quarkonium meson states represented by  $F_{1,\bar{1}}^n$ . The inverse union process does not change the situation, so we get the simplest non-trivial topological space

$$T_0 = \{\phi, F_{1,\bar{1}}, F_{1,\bar{1}}^2, \dots, F_{1,\bar{1}}^n, \dots\}, \quad (1)$$

where the subscript 0 is assigned to represent that there exist no 3-junctions in the set. However, if we include the gluonic flux-tube  $F_0$  as an excited component into  $F_{1,\bar{1}}$ , the counting rule for 3-junctions has to be changed. Pairs of incoming and outgoing 3-junctions can be added or removed indefinitely. Therefore we will not consider this possibility in this paper. Then there exists only one kind of flux-tube  $F_{1,\bar{1}}$  in the space  $T_0$ , so we may reduce the notation as

$$T_0 = \{\phi, F_{1,\bar{1}}\}, \quad (2)$$

where it is assumed that  $F_{1,\bar{1}}$  can be multiplied repeatedly without violating the law of baryon number conservation. Now, the topological space for baryon-meson system becomes

$$T_1 = \{\phi, F_3\} \quad (3)$$

with one incoming 3-junction multiplied by meson flux-tubes, which is implicitly assumed. The next baryon-meson-baryon space can be represented as

$$T_2 = \{\phi, F_3^2\}. \quad (4)$$

When outgoing 3-junctions exist, we need another index to represent the topological space, for example, the space with two incoming 3-junctions and one outgoing 3-junction can be denoted as

$$T_{2,\bar{1}} = \{\phi, F_3^2 F_{\bar{3}}, F_3 F_{2,\bar{2}}, F_{4,\bar{1}}\}. \quad (5)$$

In general, we can write down the spaces as

$$T_{i,\bar{j}} = \{\phi, F_3^i F_{\bar{3}}^j, F_3^{i-1} F_{\bar{3}}^{j-1} F_{2,\bar{2}}, \dots\}, \quad (6)$$

where  $i$  is the number of incoming 3-junctions and  $j$  is that of outgoing 3-junctions. For a given space, the baryon number  $B = i - j$  is fixed and the number of boundary points is reduced by (1,1) pair by one union operation.

For the constructed topological spaces, we can try to define physical amplitude that is appropriate for the description of gluonic degrees of freedom. The physical amplitude has to be defined in such a way that it can be used to deduce quantitative predictions about the behaviors of flux-tubes. Since the formation of a space is generated by the union and intersection operations, the amplitude can be taken to be related to the connection and disconnection of flux-tubes. One method to define the amplitude is to consider the connectedness between quarks through given flux-tube open set. Thus let's introduce the amplitude  $A$  for a quark to be connected to another quark or antiquark. We can name it as connection amplitude. If we can devise a formalism that provides quantitative descriptions of  $A$ , we may use it to predict gluonic behaviors pictured as flux-tubes.

In order to devise quantitative descriptions of  $A$ , we need to consider the relationships between  $A$  and some physically measurable quantity. Since  $A$  is defined between two quarks, one possible quantity can be chosen as the distance between the two quarks. However, the measurement of distance between quarks cannot be carried out without considering the gluonic interactions. Measurements can be done either by scattering processes or by analyzing bound state spectra. In either case, gluonic states affect the measured value, and therefore, the definition of distance between quarks has to be related in some way to gluonic properties. Since we have introduced flux-tube open sets, it is plausible to relate the measurement of distance to characteristics of open sets. As we assigned connection amplitude  $A$  to flux-tube open set, we can consider general relationships between  $A$  and measurement of distance. These relationships can be formulated by assuming the existence of a measure  $M$  of  $A$  satisfying the conditions

1.  $M(A)$  decreases as  $A$  increases,
  2.  $M(A_1) + M(A_2) = M(A_1 A_2)$  when  $A_1$  and  $A_2$  are independent.
- (7)

The measurement of distance is generalized as measure  $M$  and the first condition states that the connection probability increases for smaller measure of flux-tube. The other condition states the relation between two flux-tubes that can be joined to form single flux-tube or vice versa. With these two conditions, we can solve the measure  $M$  as functions of  $A$

$$M(A) = -k \ln \frac{A}{A_0}, \quad (8)$$

where  $A_0$  is a normalization constant and  $k$  is appropriate parameter. Now we can consider the measure  $M$  as a metric function defined between two quarks. For the simplest flux-tubes  $F_{1,\bar{1}}$ , a metric function can be introduced between the two boundary points  $\vec{x}$  and  $\vec{y}$  corresponding to the positions of a quark and an antiquark. A general form of distance function between the two points  $\vec{x}$  and  $\vec{y}$  can be written down as  $|\vec{x} - \vec{y}|^\nu$  with  $\nu$  being an arbitrary number. This distance function can be made metric for the points  $\vec{z}$  satisfying the condition

$$|\vec{x} - \vec{z}|^\nu + |\vec{z} - \vec{y}|^\nu \geq |\vec{x} - \vec{y}|^\nu. \quad (9)$$

For the points not satisfying this triangle inequality, we cannot measure the distance from boundary points with given  $\nu$ . Then it is possible to define the inner part of flux-tube as the set of points contradicting the condition in Eq. (9). With this assignment, we can figure out the shape of flux-tube and we can take  $|\vec{x} - \vec{y}|^\nu$  as an appropriate measure to deduce a concrete form for the connection amplitude  $A$ .

In general, the value of  $\nu$  is arbitrary, but the lower limit can be fixed to be 1 because there exists no point  $\vec{z}$  satisfying the triangle inequality with  $\nu < 1$ . For  $\nu = 2$  case, the shape of flux-tube becomes sphere type, and the shape changes into concave one if  $\nu > 2$ . Since we consider the gluonic flux-tube as a smooth structure, we can restrict the value of  $\nu$  as

$$1 \leq \nu \leq 2. \quad (10)$$

For a  $\nu$  value between 1 and 2, we can draw the shape of flux-tube as in Fig.2. As the value of  $\nu$  changes from 1 to 2, the flux-tube shape changes from a line into a sphere. In order to account for various possibilities, we need to sum over contributions from different  $\nu$ 's. For a small increment  $d\nu$ , the product of the two probability amplitudes for  $|\vec{x} - \vec{y}|^\nu$  and  $|\vec{x} - \vec{y}|^{\nu+d\nu}$  to satisfy the metric conditions can be taken as the probability amplitude for the increased region to be added into the inner connected region which is out of the metric condition. Then, the full connection amplitude becomes

$$A = A_0 \exp\left\{-\frac{1}{k} \int_1^\alpha F(\nu) r^\nu d\nu\right\}, \quad (11)$$



Figure 2: Flux-tube shape for  $1 < \nu < 2$  with boundaries at  $\vec{x}$  and  $\vec{y}$ .

where all possibilities from the line shape with  $\nu = 1$  to the arbitrary shape with  $\nu = \alpha$  have been included. The weight factor  $F(\nu)$  is introduced to account for possible different contributions from different  $\nu$ 's, and the variable  $r$  is

$$r = \frac{1}{l} |\vec{x} - \vec{y}| \quad (12)$$

with  $l$  being a scale parameter. If we sum up to the spherical shape flux-tube with  $\alpha = 2$ , we get in case of equal weight  $F(\nu) = 1$

$$A = A_0 \exp\left\{-\frac{1}{k} \frac{r^2 - r}{\ln r}\right\}. \quad (13)$$

In fact, we can generalize this form by replacing

$$A(r) = r^a B(r) \quad (14)$$

in Eq. (7) which is satisfied with  $a > 0$ . Then the new form of connection amplitude becomes

$$B(r) = \frac{B_0}{r^a} \exp\left\{-\frac{1}{k} \frac{r^2 - r}{\ln r}\right\}, \quad (15)$$

and we will use this form in this paper to describe the gluonic structures of tetraquarks.

### 3 Calculations of Gluonic Structures for Tetraquarks

The flux-tube overlap function  $\gamma$  represents the probability amplitude for flux-tubes to be overlapped before and after the change of flux-tube configurations [3]. It is introduced to predict the position of new quark pair which generates the changes of flux-tube boundaries. These predictions are essential in estimating the amplitudes for strong decays of hadrons. In our formalism, the function  $\gamma$  is given by the product of the initial and the final connection amplitudes

$$\gamma = A_i A_f, \quad (16)$$

where  $A_i$  and  $A_f$  represent the connection amplitudes before and after quark pair creation. Since the quark pairs are created by the gluons in hadrons, the gluonic contents of hadrons can be probed by the probability amplitudes for quark pair creations, which are described by the flux-tube overlap function  $\gamma$ . The equi- $\gamma$  curves can be taken as representing the gluonic structures of hadrons or other multiquark states.

Now let's calculate the equi- $\gamma$  curves for tetraquarks. Since the relative positions of four quarks are unknown, we consider the two cases of rectangular shape and tetrahedron shape. Rectangular configuration is typical of plane structure and tetrahedron is the simplest one in 3-dimension. For planar configuration, let's take the quark positions at  $(0,1,0)$ ,  $(0,-1,0)$ ,  $(2,-1,0)$ , and  $(2,1,0)$  as shown in Fig.3. Then the initial connection amplitude becomes

$$A_i = A_{12} A_{13} A_{14} A_{23} A_{24} A_{34}. \quad (17)$$

Since this amplitude is fixed once the quark positions are given, the overlap function  $\gamma$  is not affected by the value of  $A_i$ . However, in case of moving quarks,  $A_i$  will contribute to the final form of  $\gamma$ . In our case,  $A_i$  is just a factor of normalization. Instead the final

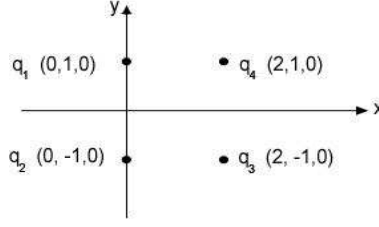


Figure 3: Four positions of quarks.

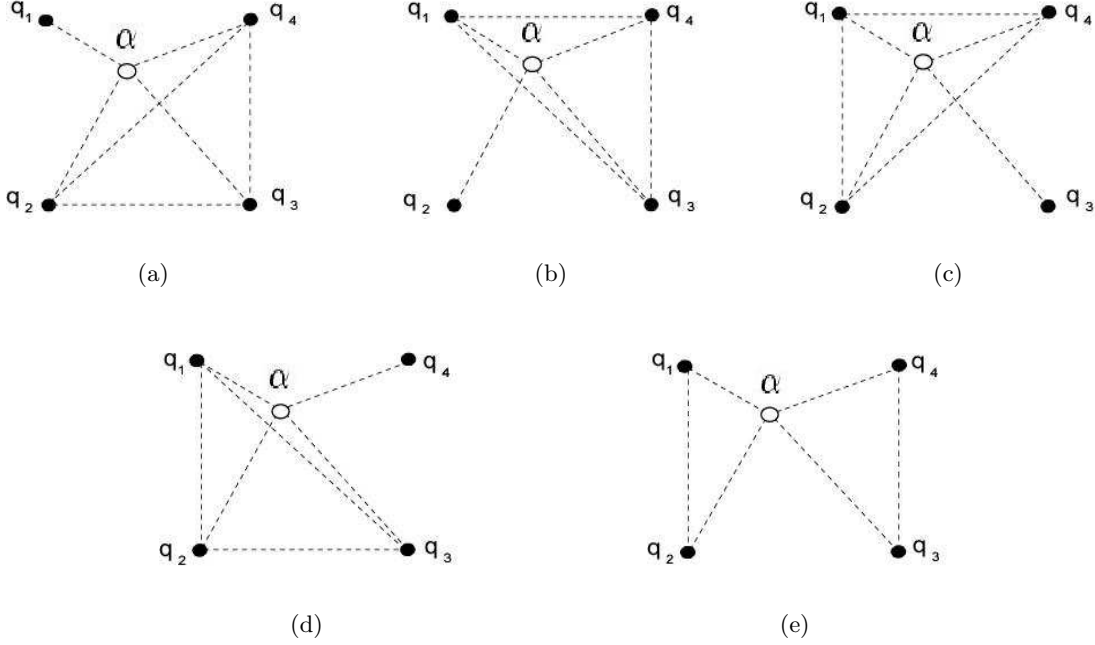


Figure 4: Possible new flux-tube connections with quark pair created at  $\alpha$ (white circle).

connection amplitude  $A_f$  depends on the position of created quark pair, and the possible flux-tube connections are shown in Fig.4.

The four cases from Fig.4(a) to Fig.4(d) correspond to one meson and one tetraquark, whereas the case of Fig.4(e) corresponds to one baryon and one antibaryon. For Fig.4(a), we can write down the  $A_f$  as

$$A_f^a = A_0^a \prod_{i=1}^4 \frac{1}{r_{\alpha i}^a} \exp \left\{ -\frac{1}{k} \frac{r_{\alpha i}^2 - r_{\alpha i}}{\ln r_{\alpha i}} \right\} \times \prod_{i,j \neq 1} \frac{1}{r_{ij}^a} \exp \left\{ -\frac{1}{k} \frac{r_{ij}^2 - r_{ij}}{\ln r_{ij}} \right\} \quad (18)$$

by using the form of connection amplitude given in Eq.(15). We can see easily that the amplitudes for Fig.4(b), (c), (d) cases are the same as in the form in Eq.(18). On the other hand, the amplitude for Fig.4(e) case becomes

$$A_f^e = A_0^e \prod_{i=1}^4 \frac{1}{r_{\alpha i}^e} \exp \left\{ -\frac{1}{k} \frac{r_{\alpha i}^2 - r_{\alpha i}}{\ln r_{\alpha i}} \right\} \times \frac{1}{r_{12}^e} \exp \left\{ -\frac{1}{k} \frac{r_{12}^2 - r_{12}}{\ln r_{12}} \right\} \times \frac{1}{r_{34}^e} \exp \left\{ -\frac{1}{k} \frac{r_{34}^2 - r_{34}}{\ln r_{34}} \right\} \quad (19)$$

with different normalization factor  $A_0^e$ . For fixed quark positions, only the first factors in Eq.(18) and Eq.(19) will contribute to the  $\gamma$  structures, and with appropriate replacements

of normalization factors we get

$$\gamma = \gamma_0 \prod_{i=1}^4 \frac{1}{r_{\alpha i}^a} \exp \left\{ -\frac{1}{k} \frac{r_{\alpha i}^2 - r_{\alpha i}}{\ln r_{\alpha i}} \right\}. \quad (20)$$

There remain two parameters  $k$  and  $a$  and we will fix the value of  $k$  as  $k = 1$  and calculate the  $\gamma$  values for several values of  $a$ .

In Fig.5, we have presented the variations of  $\gamma$  values along the  $x$ -axis. We have taken the 3 cases of  $y = 0, 0.5, 1$  for each of the  $a$  values 0.5, 1.0, and 2.0. The normalization factor  $\gamma_0$  is set to arbitrary value to get comparable graphs for different parameters. We can easily see that there appear high peaks at quark positions with non-vanishing  $a$  values. In Fig.6, equi- $\gamma$  curves are shown for three  $a$  values  $a = 0.5, 1.0$ , and 2.0. We can find that

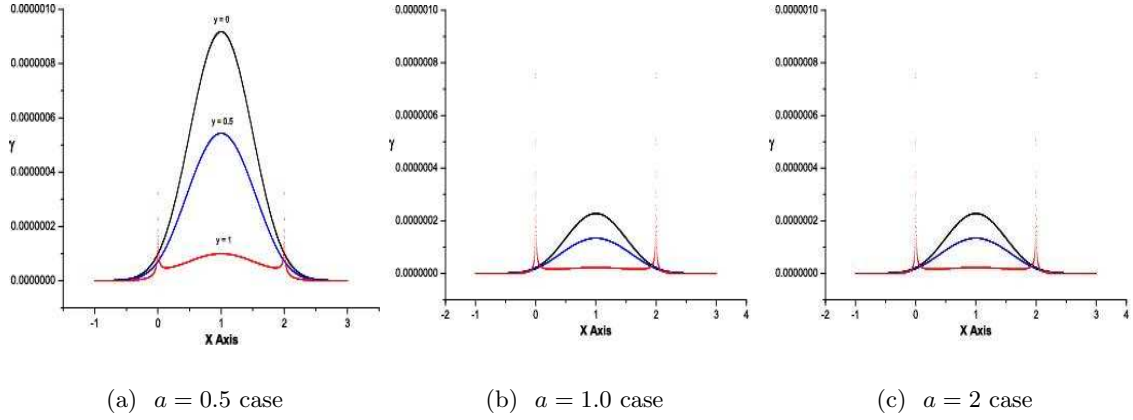


Figure 5: Calculated overlap function  $\gamma$  along the axes parallel to  $x$ -axis.

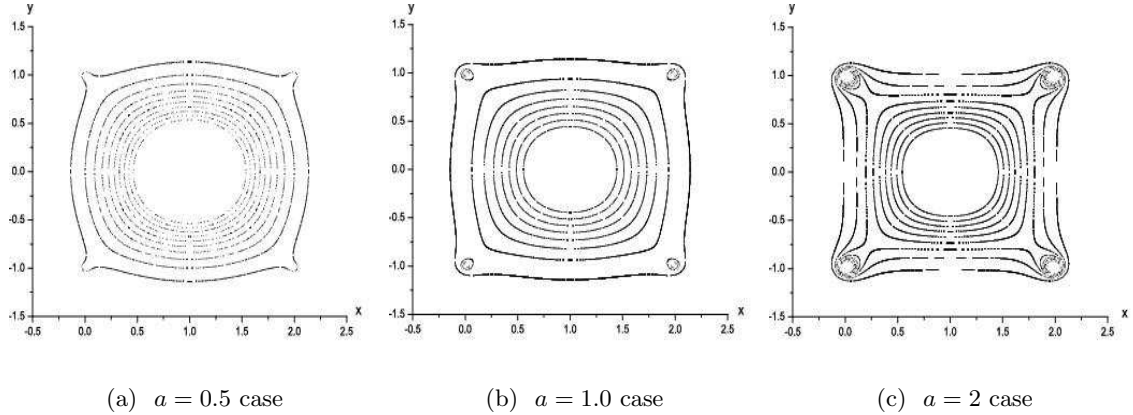
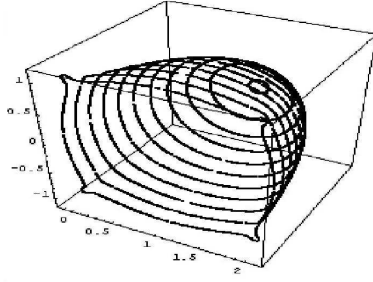


Figure 6: Equi- $\gamma$  curves for different  $a$  values.

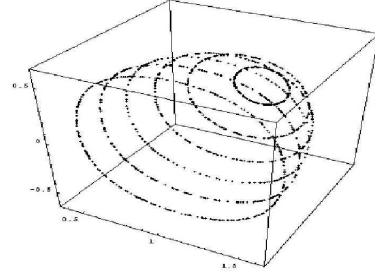
gluonic densities are higher in central region and just around the quarks. The singular behaviors at boundary quarks can be removed by introducing some cutoff radius  $r_{cut}$ , beyond which we can apply perturbative calculations factorized from nonperturbative treatment of flux-tubes. We can further draw 3-dimensional structures as in Fig.7, Fig.8, and Fig.9.

Now let's turn to the case of tetrahedron shape. If we take the four quark positions at  $(0,1,0)$ ,  $(0,-1,0)$ ,  $(\sqrt{2},0,1)$ , and  $(\sqrt{2},0,-1)$ , the situation can be drawn as in Fig.10. The



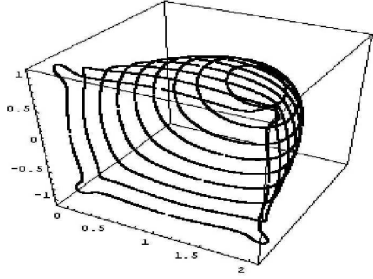


(a) Outer equi- $\gamma$  curves

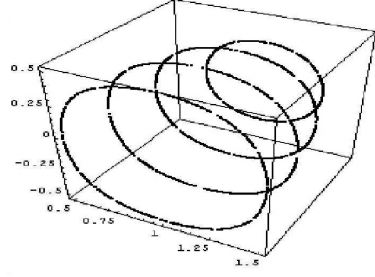


(b) Inner equi- $\gamma$  curves

Figure 7: 3-dimensional structures for  $a = 0.5$

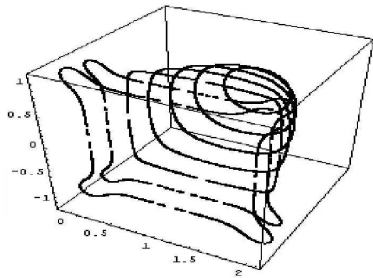


(a) Outer equi- $\gamma$  curves

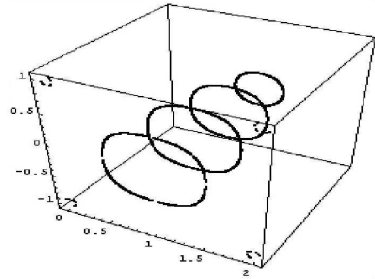


(b) Inner equi- $\gamma$  curves

Figure 8: 3-dimensional structures for  $a = 1.0$



(a) Outer equi- $\gamma$  curves



(b) Inner equi- $\gamma$  curves

Figure 9: 3-dimensional structures for  $a = 2.0$

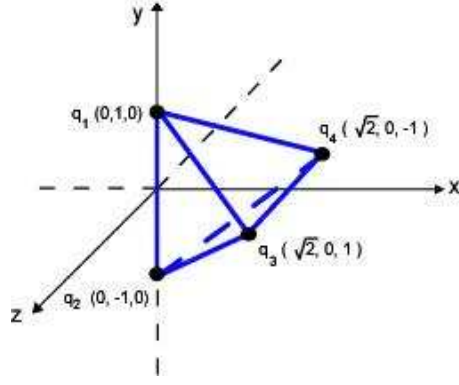


Figure 10: Four positions of quarks in tetrahedron.

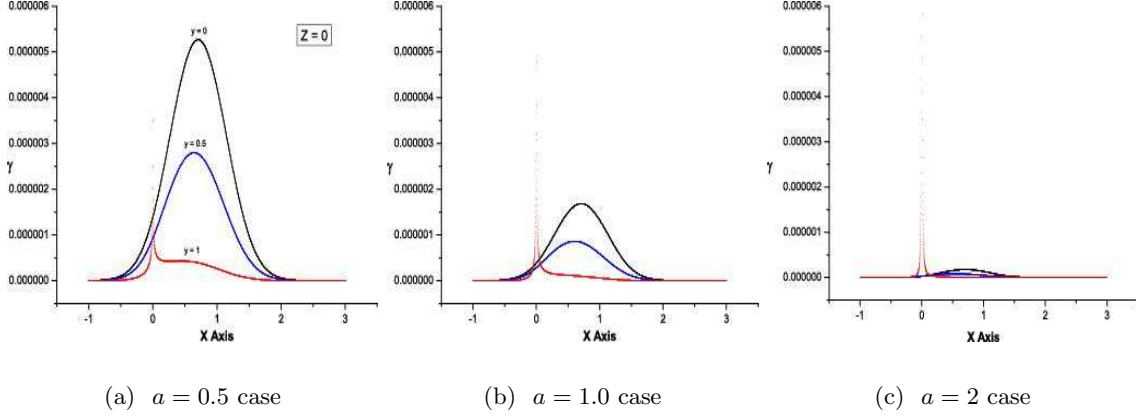


Figure 11: Flux-tube overlap function  $\gamma$  for tetrahedron shape.

flux-tube overlap function  $\gamma$  is of the same form as given in Eq.(20). With  $k = 1$ , we can calculate  $\gamma$  values for different choices of  $a$ . The results are shown in Fig.11, where  $a$  values are taken to be 0.5, 1.0, and 2.0 respectively. The three curves for each of  $a$  correspond to the  $y$  values 0, 0.5, and 1, and the  $z$  value is taken to be 0, i.e., at  $xy$  plane. At  $x = 1$ , the largest  $\gamma$  value appears at  $y = 0$ , and then it decreases as the value of  $y$  increases. These features are the same for most of  $x$  values, however, there exist exceptions around  $x = 0$  and  $y = 1$  where a boundary quark is set to be fixed. We can easily see a sharp peak around a boundary point, which can be taken as implying that gluons gather just around a quark. For general structures, we can draw the equi- $\gamma$  curves as in Fig.12. Again we can confirm the gathering of gluons around each quark, and there appears a broad peak in central part surrounded by four quarks. In order to figure out the situation in 3 dimension, we draw the equi- $\gamma$  curves in Fig.13, Fig.14, and Fig.15 with respective  $a$  values.

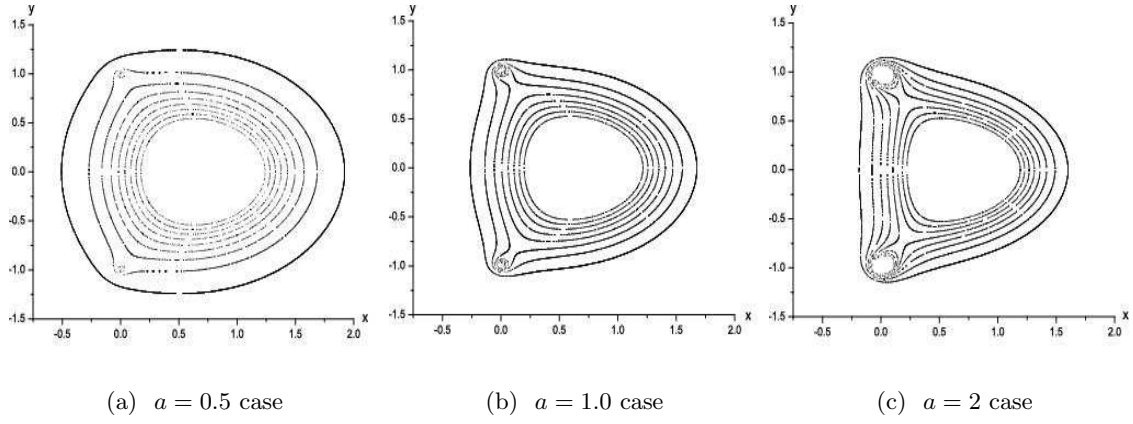


Figure 12: Equi- $\gamma$  curves in  $xy$  plane for different  $a$  values.

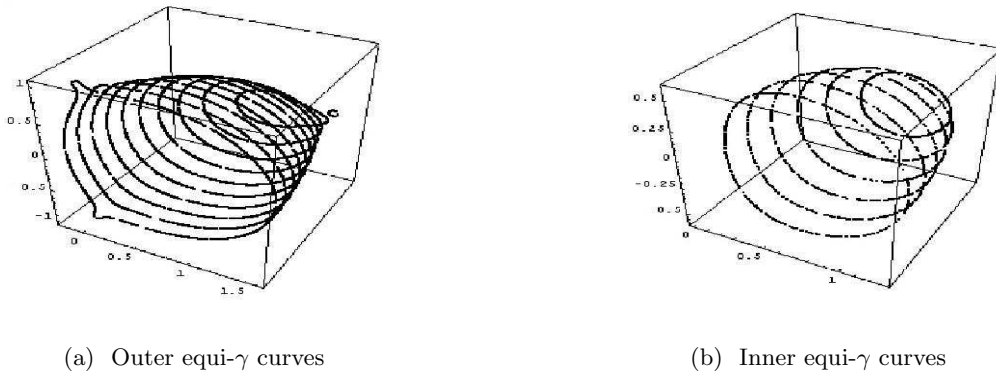


Figure 13: 3-dimensional structures of tetrahedron shape with  $a = 0.5$

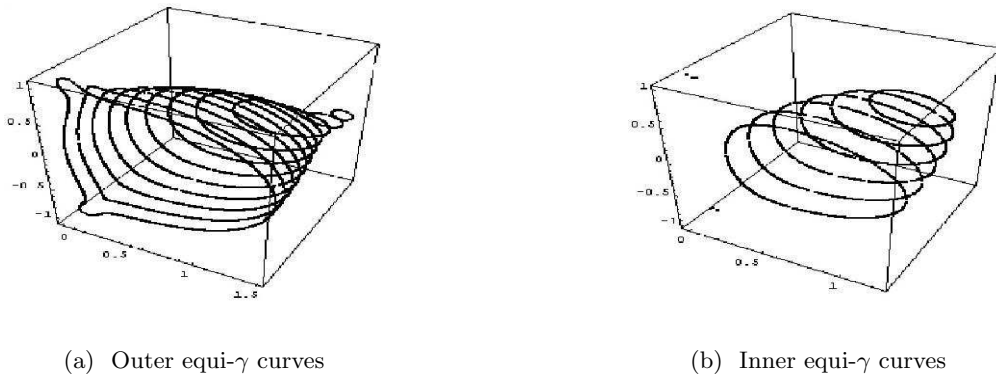


Figure 14: 3-dimensional structures with  $a = 1.0$

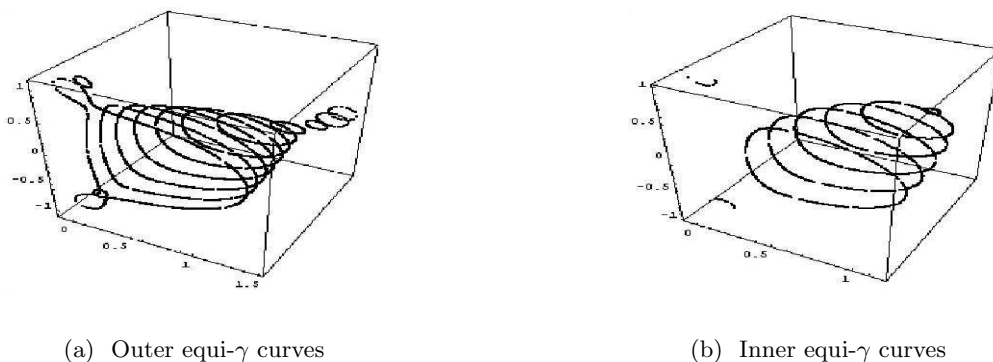


Figure 15: 3-dimensional structures with  $a = 2.0$

## 4 Discussions

In this paper, we have given a systematic formulation of flux-tubes which can be applied to estimate the gluonic structures of hadrons quantitatively. We can classify the flux-tubes and construct topological spaces with simple assumptions. For the constructed topological spaces, we can define connection amplitude which can be used to deduce flux-tube overlap functions for various configurations of boundary quarks. We have presented our results for tetraquarks which are now of special interests after the discovery of  $X(3872)$ . Since we do not know the configurations of quarks, we have carried out the calculations for two different situations. These different situations can be compared if the total gluon densities are obtained by integration, however, the relative sizes between the two situations are not known.

The extension to pentaquarks or hexaquarks is immediate in our formalism. But the most critical loophole lies in the process of fixing the positions of boundary quarks. This problem is more evident if we consider the simpler system of baryons. For example, in case of a proton, we still do not know the relative positions and motions of valence quarks [12]. In order to analyze the probed results, we have to introduce various kinds of generalized parton distributions [13] which are the main subjects discussed in recent SIR Workshop at J-lab. Up to the present knowledge, the quarks appear to have orbital angular momentum which is in some way related to the gluonic motions or structures in proton. These relations have to be studied more in the future. The spatial dependences of gluonic structures may be closely related to the concept of impact parameter dependent distributions [14].

In fact, we have defined the flux-tube overlap function  $\gamma$  as a product of the initial and the final connection amplitudes  $A_i$  and  $A_f$ . For fixed quark positions,  $A_i$  becomes a constant and therefore has no effects on the form of  $\gamma$ . Then there could be the questions about the meaning of  $A_i$  and the interpretation of  $\gamma$  as a product of two amplitudes. One possible answer can be sought by considering the initial motions of quarks or changes of relative distances between quarks. Since the quark distribution amplitudes are related to these variations,  $A_i$  may be used to represent the quark distributions. In our calculations, only the contributions of  $A_f$  have been considered and it turns out that the main gluonic contributions come from the central region surrounded by four quarks. This is the same for the two cases considered, i.e., rectangular shape and tetrahedron shape. Of course, there appear singular behaviors around boundary quarks, which have to be cutoff at some point where perturbative calculations can be applied.

As for the state  $X(3872)$ , there exist many models such as  $c\bar{c}$  [15], hybrid charmonium [16], diquark-antidiquark [17], glueball [18], cusp at  $D^0\bar{D}^0$  threshold [19], and  $D^{*0}\bar{D}^0$

molecule [20]. Of these possibilities, models of diquark-antidiquark and  $D^{*0}\bar{D}^0$  molecule correspond to tetraquarks. However, the more compact tetraquark state will be a state with equal footings of the four quarks. In our case, tetrahedron shape can be taken to be in that state. In contrast, rectangular configuration will correspond to diquark-antidiquark or  $D^{*0}\bar{D}^0$  molecule state. The discriminations of these models based on more data and more theoretical works have to be done in the future.

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